

Goniometrische vergelijkingen

1. Los de volgende vergelijkingen op:

a) $\sin\left(3x - \frac{1}{2}\pi\right) = 0$

c) $\sin\left(4x - \frac{1}{3}\pi\right) = 1$

b) $\cos\left(\frac{1}{2}x - \frac{1}{6}\pi\right) = 0$

d) $\cos(4\pi x) = -1$

2. Los de volgende vergelijkingen op (gebruik de eenheidscirkel!):

a) $2\sin\left(2x - \frac{1}{6}\pi\right) = \sqrt{2}$

c) $2\sin\left(2x - \frac{1}{4}\pi\right) = -\sqrt{3}$

b) $2\cos\left(3x - \frac{1}{2}\pi\right) = \sqrt{3}$

d) $2\cos\left(x - \frac{1}{3}\pi\right) = -1$

3. Bereken exact de oplossingen op $[0, 2\pi]$:

a) $2\sin\left(\frac{1}{2}x\right) = 1$

c) $\sin\left(\frac{2}{3}x\right) = -\frac{1}{2}\sqrt{2}$

b) $2\cos(3x - \pi) = -1$

d) $\cos(4\pi x) = -\frac{1}{2}\sqrt{3}$

4. Gegeven is de functie $f(x) = -2 + 3\sin(3x + \pi)$ met domein $[0, \pi]$.

a) Geef de coördinaten van de snijpunten met de x -as.

b) Geef de exacte coördinaten van de toppen (maxima).

c) Geef de exacte coördinaten van de dalen (minima).

5. Gegeven is de formule $T = 7,6 + 4,3\sin\left(\frac{\pi}{12}(u - 10)\right)$.

a) Bepaal voor welke u de waarde van T maximaal is. Daarbij mag je u tussen 0 en 24 kiezen.

b) Bepaal voor welke u de waarde van T minimaal is. Daarbij mag je u tussen 0 en 24 kiezen.

c) Bepaal voor welke u geldt $T = 7,6$.

d) Bepaal voor welke u geldt $T = 9,75$.

6. Los de volgende vergelijkingen algebraïsch op:

a) $\sin^2(x) - \sin(x) = 0$

i) $4\sin^2\left(3x - \frac{1}{2}\pi\right) = 1$

b) $\cos^2(2x) + \cos(2x) = 0$

j) $4\cos^2\left(x + \frac{1}{4}\pi\right) = 3$

c) $\sin^3(x) - \sin(x) = 0$

k) $2\sin^2(2x) = 1$

d) $\cos^2\left(x - \frac{1}{3}\pi\right) = 1$

l) $2\cos^2(4\pi x) = 1$

e) $\sin^2\left(3x - \frac{1}{2}\pi\right) = 1$

m) $4\sin^3(x) - \sin(x) = 0$

f) $\sin(2x)\cos(2x) + \sin(2x) = 0$

n) $2\cos^2(x) = \cos(x) + 1$

g) $\sin\left(4x - \frac{1}{3}\pi\right) = 1$

o) $\cos^2(x) - \cos(x) + \frac{1}{4} = 0$

h) $\cos(4\pi x) = -1$

p) $4\sin^2\left(\frac{1}{5}\pi x\right) = 1$

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7. Los de volgende vergelijkingen exact op:

a) $\sin(2x + \pi) = \sin\left(\frac{1}{6}\pi\right)$

b) $\cos\left(3x - \frac{2}{3}\pi\right) = \cos(\pi)$

c) $\cos(2x) = \cos(x)$

d) $\sin(2\pi x) = \sin(\pi(x - 1))$

e) $\sin\left(2x - \frac{1}{2}\pi\right) = \sin\left(x + \frac{1}{3}\pi\right)$

f) $\cos(\pi(x - 2)) = \cos\left(\frac{1}{2}\pi x\right)$

g) $\cos(2x - 1) = \cos(x + 1)$

8. Bereken exact de oplossingen op $[0, 2\pi]$:

a) $\sin\left(2x - \frac{1}{3}\pi\right) = \sin\left(x + \frac{1}{4}\pi\right)$

b) $\cos\left(3x + \frac{1}{2}\pi\right) = \cos\left(2x - \frac{1}{4}\pi\right)$

Oplossingen

1. a) $x = \frac{1}{6}\pi + k \cdot \frac{1}{3}\pi$

b) $x = \frac{4}{3}\pi + k \cdot 4\pi$

c) $x = \frac{5}{24}\pi + k \cdot \frac{1}{2}\pi$

d) $x = \frac{1}{4} + k \cdot \frac{1}{2}$

2. a) $x = \frac{5}{24}\pi + k \cdot \pi \vee x = \frac{11}{24}\pi + k \cdot \pi$

b) $x = \frac{2}{9}\pi + k \cdot \frac{2}{3}\pi \vee x = \frac{7}{9}\pi + k \cdot \frac{2}{3}\pi$

c) $x = \frac{23}{24}\pi + k \cdot \pi \vee x = \frac{19}{24}\pi + k \cdot \pi$

d) $x = \frac{13}{12}\pi + k \cdot 2\pi \vee x = \frac{19}{12}\pi + k \cdot 2\pi$

3. a) $x = \frac{1}{3}\pi \vee x = \frac{5}{3}\pi$

b) $x = \frac{1}{9}\pi \vee x = \frac{5}{9}\pi \vee x = \frac{7}{9}\pi \vee x = \frac{11}{9}\pi \vee x = \frac{13}{9}\pi \vee x = \frac{17}{9}\pi$

c) $x = \frac{15}{8}\pi$

d) $x = \frac{5}{24} \vee x = \frac{17}{24} \vee x = \frac{19}{24} \vee x = \frac{29}{24} \vee x = \frac{31}{24} \vee x = \frac{41}{24} \vee x = \frac{43}{24} \vee x = \frac{53}{24} \vee x = \frac{55}{24} \vee x = \frac{65}{24} \vee x = \frac{67}{24} \vee x = \frac{77}{24} \vee x = \frac{79}{24} \vee x = \frac{89}{24} \vee x = \frac{91}{24} \vee x = \frac{101}{24} \vee x = \frac{103}{24} \vee x = \frac{113}{24} \vee x = \frac{115}{24} \vee x = \frac{125}{24} \vee x = \frac{127}{24} \vee x = \frac{137}{24} \vee x = \frac{139}{24} \vee x = \frac{149}{24}$

4. a) (1,29; 0) en (1,85; 0)

b) $\left(\frac{\pi}{2}, 1\right)$ en $\left(\frac{7}{6}\pi, 1\right)$ en $\left(\frac{11}{6}\pi, 1\right)$

c) $\left(\frac{\pi}{6}, -5\right)$ en $\left(\frac{5}{6}\pi, -5\right)$ en $\left(\frac{9}{6}\pi, -5\right)$

5. a) $u = 16$

b) $u = 4$

c) $u = 10 + k \cdot 24 \vee u = 22 + 24k$

d) $u = 12 + 24k \vee u = 20 + 24k$

6. a) $x = k \cdot \pi \vee x = \frac{\pi}{2} + 2\pi k$

b) $x = \frac{\pi}{4} + \frac{\pi}{2}k \vee x = \frac{\pi}{2} + \pi k$

c) $x = k\pi \vee x = \frac{\pi}{2} + 2\pi k$

d) $x = \frac{\pi}{3} + \pi k$

e) $x = \frac{\pi k}{3}$

f) $x = \frac{\pi k}{2} \vee x = -\frac{\pi}{4} + \pi k$

g) $x = \frac{5}{24}\pi + \frac{\pi k}{2}$

h) $x = \frac{1}{4} + \frac{k}{2}$

i) $x = \frac{2}{9}\pi + \frac{\pi k}{3} \vee x = \frac{4}{9}\pi + \frac{\pi k}{3}$

j) $x = \frac{\pi}{12} + \pi k \vee x = \frac{5\pi}{12} + \pi k$

k) $x = \frac{\pi}{8} + \frac{\pi k}{4}$

l) $x = \frac{1}{16} + \frac{k}{8}$

m) $x = \pi k \vee x = \frac{\pi}{6} + \pi k \vee x = \frac{5\pi}{6} + \pi k$

n) $x = \pi + 2\pi k \vee x = \frac{2\pi}{3} + 2\pi k \vee x = \frac{5\pi}{3} + 2\pi k$

o) $x = \frac{\pi}{3} + 2\pi k \vee x = -\frac{\pi}{3} + 2\pi k$

p) $x = \frac{5}{6} + 5k \vee x = \frac{25}{6} + 5k$

7. a) $x = \frac{\pi}{12} + \frac{\pi k}{2} \vee x = \frac{5\pi}{12} + \frac{\pi k}{2}$

b) $x = \frac{5\pi}{9} + \frac{2\pi k}{3} \vee x = -\frac{\pi}{9} + \frac{2\pi k}{3}$

c) $x = \frac{2\pi k}{3}$

d) $x = 1 + 2k \vee x = \frac{2}{3}k$

e) $x = \frac{5\pi}{6} + 2\pi k \vee x = \frac{7\pi}{18} + \frac{2\pi k}{3}$

f) $x = 4k \vee x = \frac{4}{3} + 2k$

g) $x = 2 + 2\pi k \vee x = \frac{2\pi k}{3}$

8. a) $x = \frac{7\pi}{12} \vee x = \frac{13\pi}{36} \vee x = \frac{37\pi}{36} \vee x = \frac{61\pi}{36}$

b) $x = \frac{5\pi}{4} \vee x = \frac{7\pi}{20} \vee x = \frac{3\pi}{4} \vee x = \frac{23\pi}{20} \vee x = \frac{31\pi}{20} \vee x = \frac{39\pi}{20}$